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ABSTRACT

Some general suggestions for the teaching of mathematics study skills are provided in this paper. The suggestions offered are as follows: (1) teach the specialized mathematical vocabulary directly, (2) show students how the mathematical symbol system works, (3) give practice in understanding graphics, (4) give practice in spotting unstated assumptions, and (5) show how to make use of patterns commonly used in mathematics textbooks in developing mathematical concepts and principles. The paper also describes several approaches that may be used to help students learn problem solving techniques: these include providing students with a general heuristic, or model of attack, that they can use in all mathematics textbook assignments and helping students associate assigned problems with other similar problems. (GT)

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Reading - Study Skills in Mathematics

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Most study in mathematics focuses upon the development of specific concepts and principles. Simple teacher guidance in solving problems is valuable only in that the effort leads to student success in generalizing the concepts and principles so that they may solve other problems. As many mathematics teachers have noted, "Students who learn to solve an assigned problem but not how to solve others like it on their own are in trouble!" Teaching mathematics study skills, therefore, must involve (1) helping students understand and use the language of mathematics (that is, the specialized terminology, the symbol system, and the graphics used), and (2) showing them how to perceive and use patterns favored by authors of mathematics text books. general suggestions are given here.

1. Teach -- directly -- the specialized vocabulary. It is useful in mathematics study to review all approaches to vocabulary: through sound clues (phonics), structural clues (prefixes, suffixes, and roots), context clues, and dictionary or glossary. The latter two are especially important for students in math class.

Context clues abound in mathematics textbooks, chiefly because textbook authors deliberately plant them. Aware of the specialized nature of their technical vocabularies, authors frequently provide definitions for difficult words in parenthesis, in italics or boldfaced type, by examples, through definitions, or through graphics. In the pre-reading sessions, teachers need to call attention to new words and the ways they are used, thus reviewing previous instruction in the use of context clues. The idea of multiple meanings may be reviewed, too, by discussing those words which have a special meaning in mathematics and other meanings outside the field (for example, cone, angle, point, field, group, mean, radical, plane, degree). Before going through a problem on their own, students need to be reminded to watch for new words and old words used in new ways, list these in their notebooks, and check to discover if the author has defined them within the context of the problem.

The most skillful and best-intentioned authors cannot clear up all vocabulary difficulties in the text. Teachers need to identify "dangerous" words in advance, define them, and have students keep a record of them. Mathematics teachers need to review (and, in some cases, unfortunately, teach for the first time) use of the glossary, the index, dictionaries, and the particular textbook's system of providing definitions (in a box, in color, through bold face or italics).

Student vocabulary lists, valuable in all school subjects, are especially important in mathematics. Teachers need to encourage students to keep a separate section of their notebooks or a separate notebook for "Math Words." These should contain all the words encountered through the course, with definitions and examples of their use in context. Time should be set aside regularly

for in-class review, self-testing, and teacher-testing. As Armstrong counseled students years ago: "Make the vocabulary of mathematics the first element of study in all mathematical subjects" (Armstrong, 1967, p.99). The concepts and principles are approached only through the vocabulary, and, often, the vocabulary words themselves represent basic concepts and principles.

2. Show students how the symbol system works.-- The "language" of mathematics is more than lexical items arranged syntactically; it includes a highly-evolved, arbitrary symbol system. To "read" mathematics implies not only understanding words in sentences but, also, the understanding of systematic symbol systems. In elementary school, the symbols may be dollar signs, decimal points, equal signs, and plus or minus signs; in high school and college, the range of symbols increases as students get into algebra, geometry, and calculus. Again, it is important for students to keep records in their notebooks of the symbols they encounter in the textbook and in class lessons. Their textbooks may include often-used symbols in a glossary or separate section of the chapter, but keeping individual notebook records personalizes the learning and, to some extent, provides both students and teacher with a control over the in-take, recording, verification, and collection of the key symbols.

Experienced mathematics teachers often recommend that students, especially in elementary school, be reminded regularly that it is acceptable to read slowly and frequently shift attention from the text to their symbol lists. (See, for example, Hater, Kane, and Byrne, 1974) Some students feel obliged to move their eyes in a straight-forward manner, from left to right, across the lines, and down the page. They need to realize that in mathematics assignments it is necessary to re-read, check the meanings of words, and, especially verify their use of the symbols.

An effective way of making students, even in high school and college, aware of the meaning and interrelationships among symbols and symbol systems is through oral reading. Robinson notes,

Seemingly, the very art of verbalizing gives help in the process of clarification. This type of oral reading -- to help students focus with accuracy on the interrelationship among the symbol systems -- should be very much like the oral reading that is recommended for proof reading -- unit by unit -- but with active thinking taking place.

(Robinson, 1978, p. 235)

3. Give practice in understanding graphics. - Graphics are as much a part of the language of mathematics as the special terminology or the symbol systems. They include: figures, graphs, diagrams, tables, and charts. They need to be taught as an intrinsic part of the language because most text books in the area use them as visual explanations of verbal material. They are used regularly to introduce main points, to reinforce previous points, and often as integral parts of important concepts and principles.

Some teachers approach graphics through graphs and tables, beginning with simple bar graphs and having students develop corresponding tables to complement each graph. They start with phenomena close to the lives of students (number of books read by individuals in the class for fourth or fifth graders; scores from athletic events for secondary school students) and have some groups present the information pictorially in some type of graph and other groups present the

same information in the table. Then, they have the class compare and contrast the final products. Other teachers explain and have students create similar graphs and tables as they are encountered in the textbook. Most teachers who have successfully taught graphics make regular reference to diagrams, tables, graphs, and charts found in current newspapers and magazines.

An effective way of helping students relate pictorial and verbal presentations is to have them cover, with a card or paper, the verbal material accompanying a selected figure in their textbook and then write a verbal description of what the figure tries to communicate. After the verbal descriptions have been shared and compared with the textbook author's, students may then do the exercise in reverse, that is, cover another selected graphic and, from the book's verbal description, draw a figure. These, too, need to be shared and compared with the figure in the book. Students, generally, should be encouraged to prepare their own graphics for textbook material, especially for verbal material not pictorialized in the book. Their work should be duplicated or posted as valid contributions to all students' understanding of the textbook.

4. Give students practice in spotting "unstated assumptions." -- The matter of "unstated assumptions" has been treated by the mathematician Polya in his suggestions for helping students read mathematical problems (Polya, 1962). He presents the following "word" problem:

A patrol plane flies 220 miles per hour in still air. It carries fuel for 4 hours of safe flying. If it takes off on patrol against a wind of 20 miles per hour, how far can it fly and return safely?

Polya points out that a student may "read" these sentences in a straightforward way and proceed to solve the problem. His or her success, however, is contingent upon the recognition of at least three unstated assumptions: (1) The wind is supposed to blow with unchanged intensity during the entire flight, (2) The plane travels in a straight line, and (3) The time needed for changing directions at the furthest point is negligible. Polya notes that all word problems contain such unstated assumptions and demand from the reader preliminary interpretation and abstraction: "This," he writes, "is an essential feature of the word problem which is not always trivial and should be brought into the open" (Polya, 1965, p. 30). Such assumptions permeate mathematics textbooks from elementary school to college. Teachers need to watch for them constantly and to encourage students to be vigilant as they read. Time spent in class identifying and examining unstated assumptions helps students become better math students and may help them in all school subjects. Such efforts, too, may help them become better critical thinkers long after they have completed their mathematics courses in school.

5. Demonstrate how patterns work. -- After examining various definitions of mathematics, Sawyer settled upon: "Mathematics is the classification and study of all possible patterns" (quoted in Armstrong, 1967, p. 100). He defined a pattern, simply, as "any kind of regularity that can be recognized by the mind." The importance of recognizing and using certain simple patterns regularly used by authors of mathematics textbooks needs to be examined here.

After surveying and studying a large number of popular mathematics text books, Robinson found three major patterns favored by most authors (Robinson, 1978, pp. 240-256). Even beginning students should know these:

- Concept development pattern. Usually, the concept the author wants to develop is set off by boldface, italics, boxes, or color. The goal of the author (and the teacher using the book) is to make the concept clear to student readers by having them work out the details of a practical problem which illustrates the concept. Robinson suggests a six-step strategy: study the heading, read the definition(s), reread the definition(s), read the explanatory information and graphics, reread when necessary, and then reread the definitions (Robinson, 1978, p. 243). To this should be added: groups of problems illustrating the same concept should be worked on by the class so that students begin to see the basic concept from different points of view.

- Principle development. A mathematical principle (that is, a postulate, theorem, law, rule, or axiom) is a generalization growing out of a group of concepts. All are more-or-less formal statements of a mathematical truth or assumed truth. Usually, the author expresses this larger generalization in bold face, italics, color, or capital letters; usually, too, the author provides solutions as illustrations of the principle immediately after its statement. Robinson notes that the following strategy works for many students: study the heading and introductory material, read the principle, reread the principle, read illustrations and sample problems, reread, compute, write, and/or draw as needed, and, finally, reread the principle. As in the case of the concept development pattern, students should do problems in groups to see how a basic law, rule, or axiom is derived from a multitude of separate examples.

- Problem solving. Because problem solving, especially in form of word problems, is often most troublesome to students, teachers need to attack this "pattern" in a variety of ways. Cooney, Davis, and Henderson present a set of guidelines for teachers to follow in helping students approach word problems (Cooney, Davis and Henderson, 1975). Robinson has summarized these as follows:

1. Make sure students understand the problem.
 - a. Do students understand the meanings of terms in the problem?
 - b. Do they take all relevant information into consideration?
 - c. Can they indicate what the problem is asking them to find?
 - d. Can they state the problem in their own words?
 2. Help students gather relevant thought material to assist in creating a plan.
 - a. Assist them in gathering information by having them analyze the given conditions (and sometimes the assumed solution).
 - b. Help them obtain information by analyzing an analogous problem.
 - c. When they become discouraged by pursuing an unproductive approach, help them view the problem from a different perspective.
 3. Provide students with an atmosphere conducive to problem solving.
 4. Once students have obtained a solution, encourage them to reflect on the problem and the means of solution.
 - a. If possible have students verify solutions that have not been established deductively.
 - b. Encourage them to seek and present alternative ways of solving a problem.
 - c. Challenge them to investigate variations of the given problem.
- (Robinson, 1978, p. 252)

. Provide a heuristic. -- Robinson's summary of guidelines to problem solving leads to the important matter of providing students with a general heuristic, or model of attack, they can use in all textbook assignments in mathematics.

- Have students read through the problem, taking an over-view to give them a general notion of why the class is doing the problem and the ways in which it fits into the context of the course.
- Explain new and possibly difficult words (such as patrol plane, still air, and safe flying in the problem on page 3).
- Set up a purpose and goal for "reading" (usually the answer to the question posed in the problem; in this case: "How far can it fly and return safely?").
- Have students read the problem.
- Ask them to self-test themselves and share their answers before revealing the correct answer.

While such an approach seems better than simply assigning the problem "cold," it clearly is not enough. It is worthwhile here to seek help from the mathematicians themselves.

Polya, for example, would agree that a format, approach, or model is needed, "The solution of many problems consists of essentially a procedure, a course of action, a scheme of well-interrelated operations, a modus operandi (Polya, 1962, p. 1). He, too, suggests certain pre-reading or pre-solving activities and the explanation of possibly difficult vocabulary items. He, also, as noted above, calls attention to "unstated assumptions" within the text (see page 5). He offers a seven-step "heuristic" for all problems, easy or difficult; the student asks:

1. What do I want?
2. What are the data? (or What do I have?)
3. What is the unknown?
4. How is it defined?
5. Is there an analogous problem?
6. What do I want in terms of the analogous problem?
7. How can I get it? (Polya, 1965)

Fehr (1955) offers a five-step approach based on his belief that all learning requires association. Learning to read and solve verbal problems means, he says, that students must be able to relate or associate the assigned problem with similar ones. He suggests that students:

1. Select significant words and phrases in a situation, show their importance to the situation, and attempt to define them.
2. Attempt to identify statements and implicit assumptions essential to the conclusion.
3. Attempt to evaluate the assumptions, accepting only those that past experience makes plausible and rejecting others.
4. Require evidence to support the conclusion.
5. Apply this method in establishing a mathematics structure. (Fehr, 1955)

The approaches to classroom listening and reading given for general use in all school subjects combined with those of Robinson, Polya, Fehr, and specialists in mathematics education lead to ten suggestions for helping students read school mathematics assignments:

1. Make sure students read through the assignment first and understand what they are seeking.
2. Explain all difficult words and phrases (both mathematical terms and ordinary English words students may not know).
3. Encourage students to visualize as much as possible by using graphs, charts, diagrams, and pictures.
4. Do several easier sample exercises (either taken from the textbook or created for the purpose).
5. Associate the materials encountered in the assignment with previous learnings and other experiences from mathematics, from other school subjects, and from their own lives.

6. Call attention to basic arithmetic fundamentals involved (review and reteach if necessary).
7. Emphasize the basic principles in the problem or assignment.
8. Have students "make up" similar problems to demonstrate that they understand the principle.
9. Encourage students to teach one another the principle using their own examples and word problems.
10. Demonstrate how the homework (if any) is to be done.

A true heuristic, it must be noted, does not lie in suggestions for teachers such as these. It is, rather, an internalized procedure or approach (or to use Polya's synonyms, a "course of action, a scheme of well-interrelated operations, a modus operandi"). It is the student's, -- not the teacher's. The teacher -- and this seems true of all the study skills described in this book -- can only explain, suggest, and provide much meaningful, relevant, guided practice in the use of the heuristic or, especially in the case of more mature students, of several heuristics. The key word here is much.

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